LECTURE NO 21

Electrostatics

TOPIC COVERED

- Boundary condition between dielectric and dielectric
- Boundary condition between dielectric and conductor

So far, we have considered the existence of the electric field in a homogeneous medium. If the field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called *boundary conditions*. These conditions are helpful in determining the field on one side of the boundary if the field on the other side is known. Obviously, the conditions will be dictated by the types of material the media are made of. We shall consider the boundary conditions at an interface separating

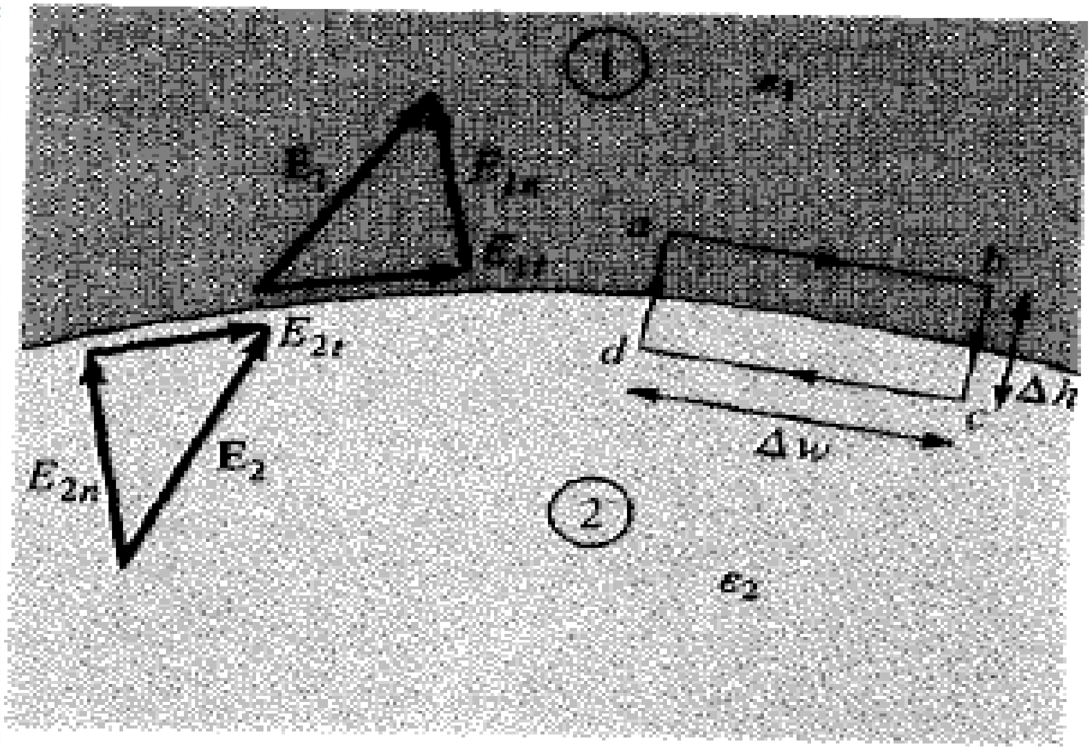
- dielectric (ε_{r1}) and dielectric (ε_{r2})
- conductor and dielectric
- conductor and free space

To determine the boundary conditions, we need to use Maxwell's equations:

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 \tag{5.52}$$

and

$$\oint \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}} \tag{5.53}$$



Consider the **E** field existing in a region consisting of two different dielectrics characterized by $\varepsilon_1 = \varepsilon_o \varepsilon_{r1}$ and $\varepsilon_2 = \varepsilon_o \varepsilon_{r2}$ as shown in Figure 5.10(a). $\mathbf{E}_1$ and $\mathbf{E}_2$ in media 1 and 2, respectively, can be decomposed as

$$\mathbf{E}_1 = \mathbf{E}_{1t} + \mathbf{E}_{1n} \tag{5.55a}$$

$$\mathbf{E}_2 = \mathbf{E}_{2t} + \mathbf{E}_{2n} \tag{5.55b}$$

We apply eq. (5.52) to the closed path *abcda* of Figure 5.10(a) assuming that the path is very small with respect to the variation of **E**. We obtain

$$0 = E_{1t}\,\Delta w - E_{1n}\frac{\Delta h}{2} - E_{2n}\frac{\Delta h}{2} - E_{2t}\,\Delta w + E_{2n}\frac{\Delta h}{2} + E_{1n}\frac{\Delta h}{2} \tag{5.56}$$

where $E_t = |\mathbf{E}_t|$ and $E_n = |\mathbf{E}_n|$. As $\Delta h \to 0$, eq. (5.56) becomes

$$\boxed{E_{1t} = E_{2t}} \tag{5.57}$$

Thus the tangential components of E

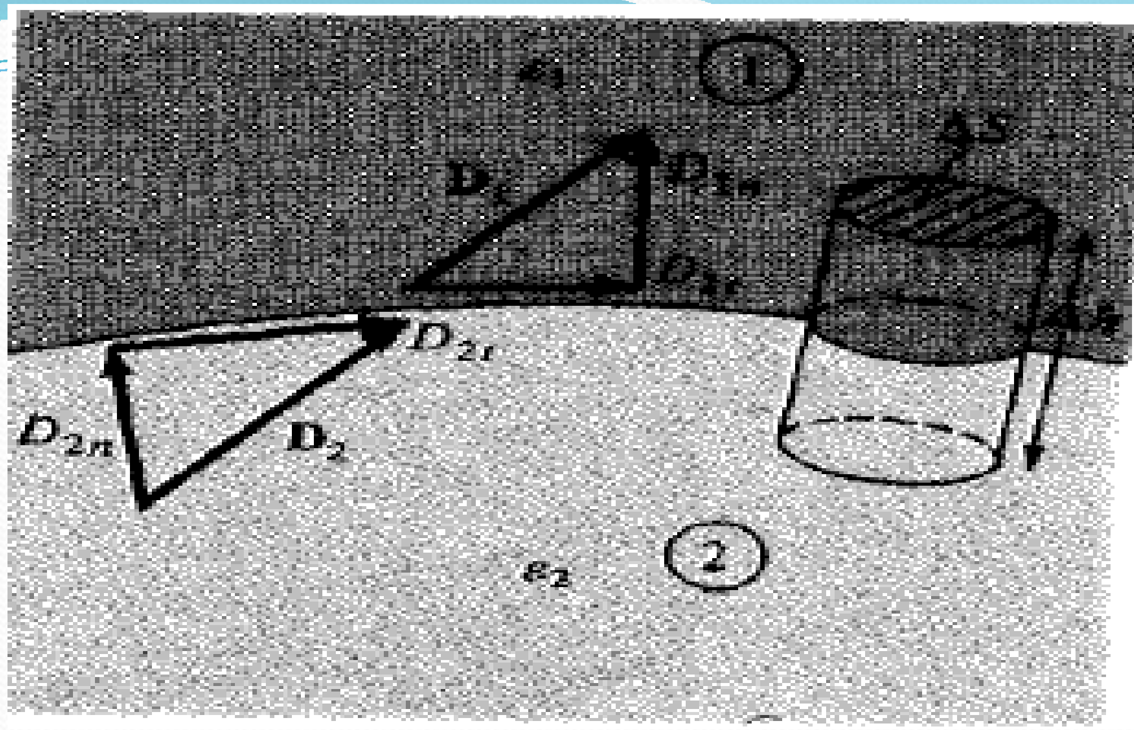
Thus the tangential components of **E** are the same on the two sides of the boundary. In other words, $\mathbf{E}_t$ undergoes no change on the boundary and it is said to be *continuous* across the boundary. Since $\mathbf{D} = \varepsilon\mathbf{E} = \mathbf{D}_t + \mathbf{D}_n$, eq. (5.57) can be written as

$$\frac{D_{1t}}{\varepsilon_1} = E_{1t} = E_{2t} = \frac{D_{2t}}{\varepsilon_2}$$

or

$$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2} \tag{5.58}$$

that is, D_t undergoes some change across the interface. Hence D_t is said to be *discontinuous* across the interface.



Similarly, we apply eq. (5.53) to the pillbox (Gaussian surface) of Figure 5.10(b). Allowing $\Delta h \to 0$ gives

$$\Delta Q = \rho_S \, \Delta S = D_{1n} \, \Delta S - D_{2n} \, \Delta S$$

or

$$\boxed{D_{1n} - D_{2n} = \rho_S} \tag{5.59}$$

where ρ_S is the free charge density placed deliberately at the boundary. It should be borne in mind that eq. (5.59) is based on the assumption that **D** is directed from region 2 to region 1 and eq. (5.59) must be applied accordingly. If no free charges exist at the interface (i.e., charges are not deliberately placed there), $\rho_S = 0$ and eq. (5.59) becomes

$$\boxed{D_{1n} = D_{2n}} \tag{5.60}$$

Thus the normal component of **D** is continuous across the interface; that is, D_n undergoes no change at the boundary. Since $\mathbf{D} = \varepsilon\mathbf{E}$, eq. (5.60) can be written as

$$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} \tag{5.61}$$

showing that the normal component of **E** is discontinuous at the boundary. Equations (5.57) and (5.59), or (5.60) are collectively referred to as *boundary conditions;* they must be satisfied by an electric field at the boundary separating two different dielectrics.